# Ordered Iterative Methods for Low-Complexity Massive MIMO Detection

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# Contents

# 1 Background

## Importance of MIMO

- Function: significantly improve the spectral and energy efficiency
- Role: a key pillar in 5G communication networks

with the growing number of antennas at the both sides of MIMO systems $N_r \geq N_t$

## Challenge upon the Uplink Signal Detection

### Linear Detection (ZF and MMSE)

- Advantage:

  achieve near-optimal ML detection performance
- Disadvantage:

  still with **computational complexity** $\mathcal{O}\left(N_t^3\right)$

### Sub-optimal Iterative Detectors

- Jacobi
- Gauss-Seidel (GS)
- Successive Over Relaxation (SOR)

?

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## Deep Learning (DL)

**Combine the internal structure of certain model-based algorithms with the remarkable power of deep neural networks (DNN)**

### Data-driven DL detectors

- **DetNet**
- unfold projected gradient descent via DNN
- achieve better performance than MMSE detector

### Model-driven DL detectors

- **OAMPNet**
- exploit full domain knowledge with a few trainable parameters optimized
- outperform the original OAMP

**Achieve comparable performance via DL in massive MIMO detection**

# 2 System model

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**System Model** $\mathbf{y}=\mathbf{H}\mathbf{x}+\mathbf{n}$

## Detection

**Optimal ML**

$$\hat{\mathbf{x}}=\underset{\mathbf{x}\in\mathcal{A}^{N_t}}{\operatorname{argmin}}\|\mathbf{y}-\mathbf{H}\mathbf{x}\|^2$$

Exponentially increased complexity

**Linear Detection**

$$\tilde{\mathbf{x}}_{\text{MMSE}}=\left(\mathbf{H}^H\mathbf{H}+\sigma_n^2\mathbf{I}\right)^{-1}\mathbf{H}^H\mathbf{y} \quad \hat{\mathbf{x}}_{\text{MMSE}}=Q(\tilde{\mathbf{x}}_{\text{MMSE}})\in\mathcal{A}^{N_t}$$

To bypass the matrix inversion

$$\mathbf{b}=\mathbf{H}^H\mathbf{y} \qquad \mathbf{A}=\mathbf{H}^H\mathbf{H}+\sigma_n^2\mathbf{I}$$

$$\mathbf{A}\mathbf{x}=\mathbf{b}$$

**Iterative methods are proposed to solve this linear equation**

**Jacobi**

$$x_i^{(t+1)} = x_i^{(t)} + \frac{1}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(t)} - \sum_{j=i}^{n} a_{ij} x_j^{(t)}\right)$$

**Gauss-Seidel (GS)**

$$x_i^{(t+1)} = x_i^{(t)} + \frac{1}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(t+1)} - \sum_{j=i}^{n} a_{ij} x_j^{(t)}\right)$$

**Successive Over Relaxation (SOR)**

$$x_i^{(t+1)} = x_i^{(t)} + \frac{\omega}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(t+1)} - \sum_{j=i}^{n} a_{ij} x_j^{(t)}\right)$$

**Parallel Structure**

**Conventionally in a Sequential Order ?**

**For a better presentation**

## Update Component

$$\mathbf{x}^{(t+1)} = \mathbf{x}^{(t)} + \frac{\omega}{a_{ii}} r_i^{(t)} \mathbf{e}_i, \quad r_i^{(t)} = b_i - \mathbf{a}_i^H \mathbf{x}^{(t)}$$

**Iterative** → **underlying error propagation**

## A. *Ordering Iterative Detection*

**Criterion: suppress the effect of noise**

$$\mathbf{x}^{(t+1)} = \mathbf{x}^{(t)} + \frac{\omega}{a_{ii}}(b_i - \mathbf{a}_i^H \mathbf{x}^{(t)})\mathbf{e}_i$$

$$= \mathbf{x}^{(t)} - \frac{\omega}{a_{ii}}\mathbf{a}_i^H \mathbf{x}^{(t)}\mathbf{e}_i + \frac{\omega}{a_{ii}} b_i \mathbf{e}_i$$

$$\mathbf{b} = \mathbf{H}^H\mathbf{y} = \mathbf{H}^H(\mathbf{H}\mathbf{x}+\mathbf{n}) = \mathbf{H}^H\mathbf{H}\mathbf{x} + \mathbf{H}^H\mathbf{n}$$

$$\mathbf{x}^{(t+1)} = \mathbf{x}^{(t)} - \frac{\omega}{a_{ii}}\mathbf{a}_i^H \mathbf{x}^{(t)}\mathbf{e}_i + \frac{\omega}{a_{ii}}\mathbf{g}_i^H \mathbf{x}^{(t)}\mathbf{e}_i + \frac{\omega}{a_{ii}}(\mathbf{h}_i)^H \mathbf{n}\mathbf{e}_i$$

noise part

$$\left\|\frac{\omega}{a_{ii}}(\mathbf{h}_i)^H\mathbf{n}\right\|^2 \le \omega^2 \frac{\|\mathbf{h}_i\|^2}{|a_{ii}|^2}\|\mathbf{n}\|^2$$

$$= \omega^2\|\mathbf{n}\|^2 \frac{\left|(\mathbf{h}_i)^H\mathbf{h}_i\right|}{|a_{ii}|^2}$$

$$\mathbf{A} = \mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I} \quad (a)$$

$$= \omega^2\|\mathbf{n}\|^2 \frac{\left|a_{ii} - \sigma_n^2\right|}{|a_{ii}|^2}$$

$$\propto \frac{1}{|a_{ii}|}$$

**Update in the descending order of $|a_{ii}|$**

**Restriction on OID:** **fixed order ; cyclic traversal iteration**

## B. *Modified Ordered Iterative Detection*

**Dynamic Ordering Strategy**

$$\mathbf{x}^{(t+1)} - \mathbf{x}^{(t)} = \Delta\mathbf{x} = \frac{\omega}{a_{ii}} r_i^{(t)} \mathbf{e}_i, \quad r_i^{(t)} = b_i - \mathbf{a}_i^H \mathbf{x}^{(t)}$$

**A large size Δx**

↓

**A large change of the iteration**

↓

**Positive impact upon the convergence**

**Update in the descending order of** $\left|\frac{r_i^{(t)}}{a_{ii}}\right|$

# 3 Ordering Iterative Detection Scheme

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## C. Complexity Analysis

**Algorithm 1:** OID

**Input** $\mathbf{A} = \mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}, \mathbf{b} = \mathbf{H}^H\mathbf{y}, \mathbf{x}^{(0)} = \mathbf{0}, T$
**Output** near MMSE detection solution $\widehat{\mathbf{x}}^{(t)}$

1: **for** $t = 0, 1, ..., T-1$ **do**
2: Select $i$ coordinate in descending order of $|a_{ii}|$
3: Update $\mathbf{x}^{(t+1)} = \mathbf{x}^{(t)} + \frac{\omega}{a_{ii}} r_i^{(t)} \mathbf{e}_i$
4: **end for**
5: output $\widehat{\mathbf{x}}^{(t)}$ by rounding $\mathbf{x}^{(t)}$ based on constellation $\mathcal{A}^{N_t}$

**Algorithm 2:** MOID

**Input** $\mathbf{A} = \mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}, \mathbf{b} = \mathbf{H}^H\mathbf{y}, \mathbf{x}^{(0)} = \mathbf{0}, T$
**Output** near MMSE detection solution $\widehat{\mathbf{x}}^{(t)}$

1: **for** $t = 0, 1..., T-1$ **do**
2: Update the descending order $o(i)$ by $|\frac{r_i^{(t)}}{a_{ii}}|$ when $t = 0, N_t, 2N_t, ...$
3: Select $i$ coordinate in descending order of $o(i)$
4: Update $\mathbf{x}^{(t+1)} = \mathbf{x}^{(t)} + \frac{\omega}{a_{ii}} r_i^{(t)} \mathbf{e}_i$
5: **end for**
6: output $\widehat{\mathbf{x}}^{(t)}$ by rounding $\mathbf{x}^{(t)}$ based on constellation $\mathcal{A}^{N_t}$

**Preprocess operation, reduce the complexity**

**Loosely dynamic ordering strategy**

TABLE I
COMPUTATIONAL COMPLEXITY OF ITERATIVE DETECTION SCHEMES

| MIMO detection | Multiplication | Summation |
|---|---|---|
| SOR [12] | $N_t^2 + N_t$ | $N_t^2 - N_t$ |
| OID | $N_t^2 + 2N_t$ | $N_t^2 - N_t$ |
| MOID | $2N_t^2 + 3N_t$ | $N_t^2 - N_t$ |

**OID and MOID maintain $\mathcal{O}\left(N_t^2\right)$**

# 4 MOID-Net

$$\omega_{opt} = \frac{2}{1+\sqrt{1-\rho^2(J)}}, \rho(J) = \left(1+\sqrt{\frac{N_t}{N_r}}\right)^2 - 1$$

**Not Optimal**

**For a better detection performance, upgrade the proposed MOID algorithm with DNN**

## Projection Operation

$$\mathbf{x}_k^{(T)} = \mathbf{x}_k^{(T-1)} + \frac{\omega_k}{a_{ii}} r_i^{(T)} \mathbf{e}_i$$

$$\mathbf{z}_k = \text{ReLU}(\mathbf{W}_z \mathbf{x}_k^{(T)} + \mathbf{p}_z)$$

$$\mathbf{x}_{oh,k+1} = \mathbf{W}_x \mathbf{z}_k + \mathbf{p}_x$$

$$\mathbf{x}_{k+1}^{(0)} = f_{oh}(\mathbf{x}_{oh,k+1})$$

## Learning parameters

$$\theta = \{\mathbf{W}_z, \mathbf{W}_x, \mathbf{p}_z, \mathbf{p}_x, \omega_k\}$$

## Trained by minimizing MSE loss function

$$l(\mathbf{x};\hat{\mathbf{x}}) = \sum_{k=1}^{K} log(k) \| \mathbf{x}_l - \hat{\mathbf{x}}_k \|^2$$

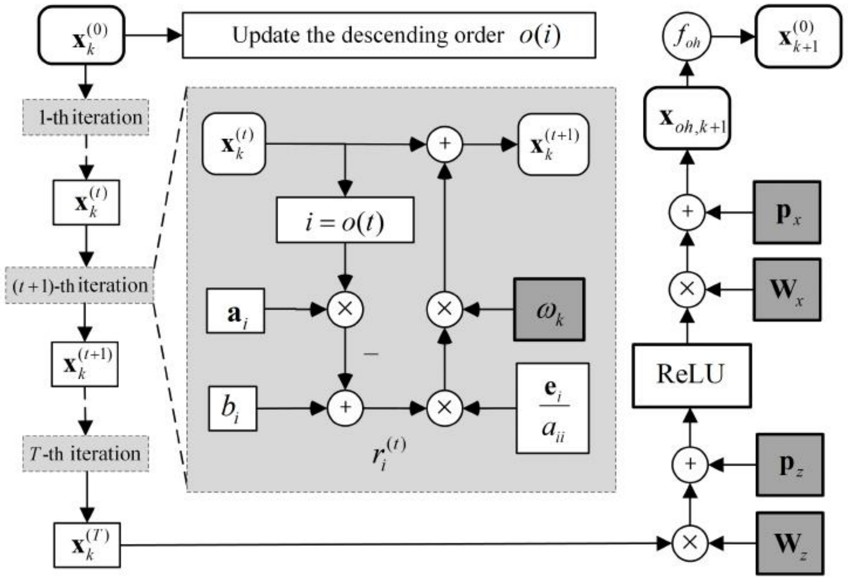


Fig. 1. The architecture of the MOID-Net detector.

# 5 Simulations

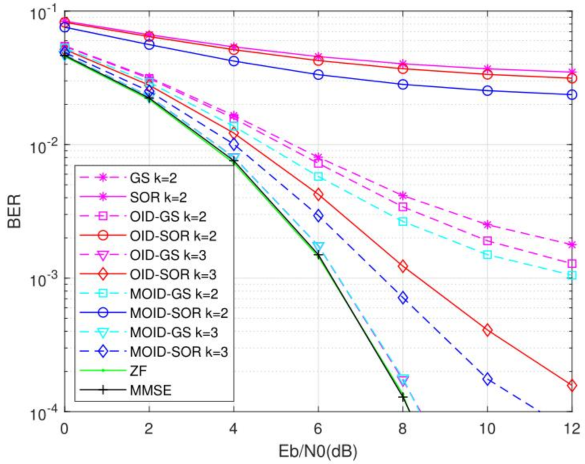


Fig. 2. Performance comparison under 64-QAM scheme of size 128 × 16.

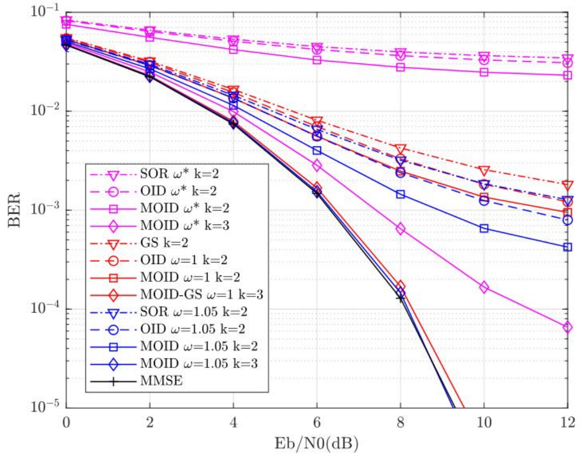


Fig. 3. Performance comparison under 64-QAM scheme of size 128 × 16 with different $\omega$.

## Detection Performance

**MOID > OID > Conventional Iterative Method**

- MMSE detection is applied as the **baseline**
- ω has a great effect on the convergence

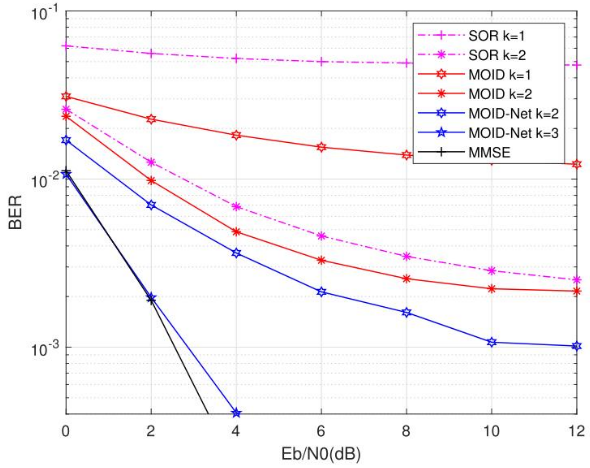


Fig. 4. Performance comparison under 16-QAM scheme of size 128 × 16.

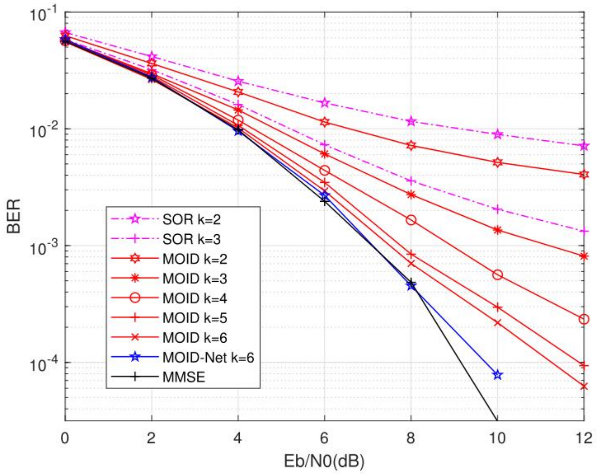


Fig. 5. Performance comparison under 4-QAM scheme of size 32 × 16.

**MOID-Net > MOID > Conventional Iterative Method**

- More iteration numbers needed with the loss of receive diversity

# Conclusion

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**Propose two ordered iterative detection methods for better signal detection performance in massive MIMO systems.**

## OID

- To **reduce error propagation** in the traditional iterative detection schemes with sequential order
- Achieves a **better detection performance** with **low complexity**

## MOID

- Convergence chiefly depends on the **residual component** during the iterations.
- A **dynamic** ordering strategy
- Further **performance improvement**

## MOID-Net

- Extend the proposed algorithm via **DNN** and **relaxation factor are trained** to optimal
- Further **performance gain**

# Thank you for your watching

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